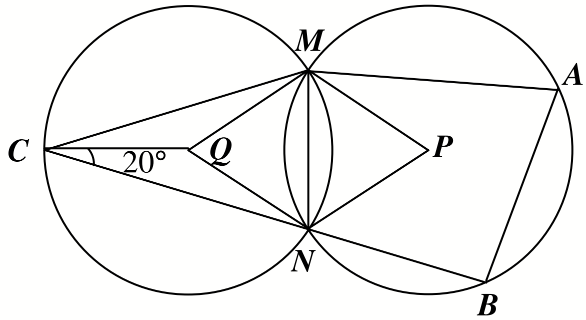
Question 1 (Basic properties of circle) [圓的基本性質]

In the figure, P and Q are the centre of the two equal circles $ABNM$ and MNC respectively. If $\angle QCN = 20°$, $AB = AM$, $MC = NC$ and BNC is a straight line, find $\angle ABN$.

[圖中，P 及 Q 分別為兩個相等圓形 $ABNM$ 及 MNC 的圓心。若 $\angle QCN = 20°$、$AB = AM$、$MC = NC$ 及 BNC 為一直線，求 $\angle ABN$。]

A. 50°
B. 90°
C. 95°
D. 105°

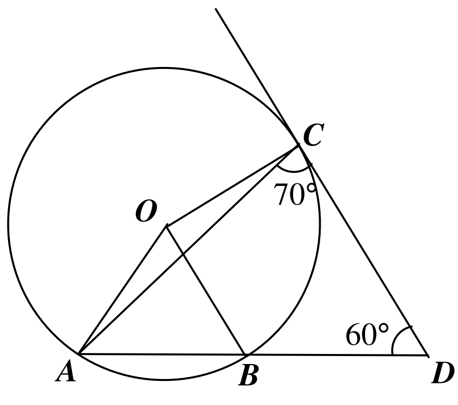


Question 2 (Basic properties of circle) [圓的基本性質]

In the figure, O is the centre of the circle. DC is the tangent to the circle at C and AD cuts the circle at B. If $\angle BDC = 60°$ and $\angle ACD = 70°$, find $\angle AOB$.

[圖中，O 為圓形的圓心。DC 為與 C 圓相切於的切線及 AD 與圓相交於 B。若 $\angle BDC = 60°$ 及 $\angle ACD = 70°$，求 $\angle AOB$。]

A. 40°
B. 50°
C. 60°
D. 70°



Question 3 (Permutation and Combination) [排列與組合]

There are 30 circles of different size. What is the maximum number of intersection points they can formed?

[有 30 個不同大小的圓形。它們可組成的交點數目最多有多少個？]

A. 30
B. 435
C. 870
D. 1740

Question 4 (Complex Number) [複數]

Let $z=(k-10)i-\dfrac{(k+4)i}{3-i}$, where k is a real number. If z is a real number, find $\dfrac{k}{z}$.

[設 $z=(k-10)i-\dfrac{(k+4)i}{3-i}$ ，其中 k 為一實數。若 z 為一實數，求 $\dfrac{k}{z}$ 。]

A. -8
B. 8
C. 10
D. 16

Question 5 (Quadratic Equation) [二次方程]

Let a be a constant. If α and β are the roots of the equation $x^2-4x+a=0$, then $2\alpha^2+8\beta=$

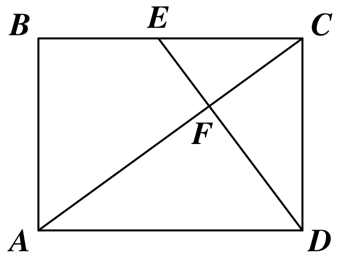
[設 a 為常數。若 α 及 β 為方程 $x^2-4x+a=0$ 的根，則 $2\alpha^2+8\beta=$]

A. $16-a$
B. $16+a$
C. $32-2a$
D. $32+2a$

Question 6 (Mensuration) [求積法]

The figure shows a rectangle $ABCD$. E is a point on BC such that AC and ED intersects at F. If $BE = CE$, find Area of ΔFCD : Area of $ABEF$.

[圖中所示為一長方形 $ABCD$。E 為 BC 上的一點使得 AC 與 ED 相交於 F。若 $BE = CE$，求 ΔFCD 的面積 : $ABEF$ 的面積。]



A. $1:3$

B. $2:3$

C. $2:5$

D. $4:7$

Question 7 (Coordinate Geometry) [坐標幾何]

$M(-4,\ 2)$ and $N(-10,-8)$ are two points. It is given that K is a point lying on the straight line $y = x + 20$ such that $MK = NK$. Find the y-coordinate of K.

[$M(-4,\ 2)$ 及 $N(-10,-8)$ 為兩點。已知 K 為直線 $y = x + 20$ 上的一點使得 $MK = NK$。求 K 的 y 坐標。]

A. -17

B. -3

C. 3

D. 17

Question 8 (Equation of Straight Line) [直線方程]

Find the equation of perpendicular bisector of the points $A(3,\ 2)$ and $B(7,-4)$.

[求點 $A(3,\ 2)$ 及 $B(7,-4)$ 的垂直平分線的方程。]

A. $2x - 3y = 0$

B. $2x - 3y - 13 = 0$

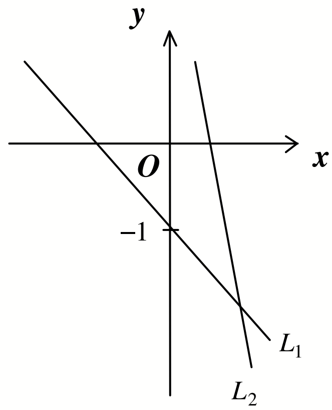
C. $2x - 3y - 26 = 0$

D. $2x + 3y - 7 = 0$

Question 9 (Equation of Straight Line) [直線方程]

In the figure, the equations of the straight lines L_1 and L_2 are $x+my=n$ and $x+py=q$ respectively. Which of the following are true?

[圖中，直線 L_1 及直線 L_2 的方程分別為 $x+my=n$ 及 $x+py=q$。下列何者正確？]



I. $m>p$

II. $n<q$

III. $n+m<p+q$

A. I and II only [只有 I 及 II]

B. I and III only [只有 I 及 III]

C. II and III only [只有 II 及 III]

D. I, II and III [I、II 及 III]

Question 10 (Polynomials) [多項式]

Dick Hui sleeps at 03:00 and wakes up at 08:00 everyday. Now it is 06:30. Using the remainder theorem, determine whether Dick Hui is sleeping or awakening after $25^{9394}+1$ hours.

[Dick Hui 每日於 03:00 睡覺而於 08:00 起床。現在為 06:30。利用餘式定理，判斷 Dick Hui 於 $25^{9394}+1$ 小時後為睡覺中或清醒中。]

Suggested Solution [建議題解]

1. The answer is C

$\Delta QCN \cong \Delta QCM$

$\angle QCN = \angle QCM = 20°$

$\angle MQN = 2\angle QCM = 2(20°) = 80°$

$MPNQ$ is a rhombus. [$MPNQ$ 為一菱形。]

$\angle MPN = \angle MQN = 80°$

$\angle MBN = \dfrac{\angle MPN}{2} = 40°$

$\angle MNC = \dfrac{180° - \angle MCN}{2} = 70°$

$\angle MAB = \angle MNC = 20° + 50° = 70°$

$\angle MBA = \dfrac{180° - \angle MAB}{2} = 55°$

$\angle ABN = \angle MBA + \angle MBN = 40° + 55° = 95°$

2. The answer is A

$\angle CAD = 180° - 70° - 60° = 50°$

$\angle OCD = 90°$

$\angle OCA = 90° - 70° = 20°$

$\angle OCA = \angle OAC = 20°$

$\angle AOB$

$= 180° - (2)(\angle OAC + \angle CAD)$

$= 180° - (2)(20° + 50°)$

$= 40°$

3. The answer is C

Maximum number of intersection point [最多交點數目]

$= C_2^{30} \times 2$

$= 870$

4. The answer is B

$$z=(k-10)i-\frac{(k+4)i}{3-i}$$
$$z=(k-10)i-\frac{(k+4)i}{3-i}\times\frac{3+i}{3+i}$$
$$z=(k-10)i-\frac{3(k+4)i+(k+4)i^2}{9-i^2}$$
$$z=\frac{k+4}{10}+\frac{(7k-112)i}{10}$$

If z is a real number [若 z 為一實數],

$$7k-112=0$$
$$k=16$$
$$\frac{k}{z}$$
$$=\frac{16}{\frac{16+4}{10}+\frac{[7(16)-112]i}{10}}$$
$$=8$$

5. The answer is C

$\alpha+\beta=4,\ \alpha\beta=a$

Since α is a root of the equation [因 α 為方程的其中一個根],

$$\alpha^2-4\alpha+a=0$$
$$\alpha^2=4\alpha-a$$

$$2\alpha^2+8\beta$$
$$=2(4\alpha-a)+8\beta$$
$$=8\alpha+8\beta-2a$$
$$=8(\alpha+\beta)-2a$$
$$=8(4)-2a$$
$$=32-2a$$

6. The answer is C

Let area of ΔFEC [設 ΔFEC 的面積] $= k$

$\Delta ADF \sim \Delta CEF$ (AAA)

$\therefore \dfrac{\text{Area of } \Delta ADF\ [\Delta ADF \text{ 的面積}]}{\text{Area of } \Delta FEC\ [\Delta FEC \text{ 的面積}]} = \left(\dfrac{2}{1}\right)^2$

Area of ΔADF [ΔADF 的面積] $= 4\times$ Area of ΔFEC [ΔFEC 的面積] $= 4k$

$\dfrac{\text{Area of } \Delta FCD\ [\Delta FCD \text{ 的面積}]}{\text{Area of } \Delta AFD\ [\Delta AFD \text{ 的面積}]} = \dfrac{AF}{FC}$

$\dfrac{\text{Area of } \Delta FCD\ [\Delta FCD \text{ 的面積}]}{4k} = \dfrac{1}{2}$

Area of ΔFCD [ΔFCD 的面積] $= 2k$

Area of ΔACD [ΔACD 的面積] $= 2k + 4k = 6k$

$\because \Delta ABC \cong \Delta CDA$ (SSS)

$\therefore$ Area of ΔFCD [ΔFCD 的面積] : Area of $ABEF$ [$ABEF$ 的面積]

$= 2k : 6k - k$

$= 2 : 5$

7. The answer is C

$y = x + 20$

$x = y - 20$

Let [設] $K = (y - 20,\ y)$

$MK = NK$

$\sqrt{(y-20+4)^2 + (y-2)^2} = \sqrt{(y-20+10)^2 + (y+8)^2}$

$(y-16)^2 + (y-2)^2 = (y-10)^2 + (y+8)^2$

$2y^2 - 36y + 260 = 2y^2 - 4y + 164$

$y = 3$

8. The answer is B

Slope of AB [AB 的斜率] $= \dfrac{2-(-4)}{3-7} = -\dfrac{3}{2}$

Slope of perpendicular bisector [垂直平分線的斜率] $= \dfrac{2}{3}$

Mid point of AB [AB 的中點] $= \left(\dfrac{3+7}{2}, \dfrac{2+(-4)}{2}\right) = (5, -1)$

Equation of perpendicular bisector [垂直平分線的方程]: $\dfrac{y-(-1)}{x-5} = \dfrac{2}{3}$

$2x - 3y - 13 = 0$

9. The answer is D

$L_1: x+my=n \Rightarrow y=-\frac{1}{m}x+\frac{n}{m}$

From the graph [由圖中所見], slope of L_1 [L_1 的斜率] <0

$-\frac{1}{m}<0 \Rightarrow m>0$

y-intercept of L_1 [L_1 的 y 截距] $=-1$

$\frac{n}{m}=-1 \Rightarrow n=-m$

$L_2: x+py=q \Rightarrow y=-\frac{1}{p}x+\frac{q}{p}$

From the graph [由圖中所見], slope of L_2 [L_2 的斜率] <0

$-\frac{1}{p}<0 \Rightarrow p>0$

y-intercept of L_2 [L_2 的 y 截距] >0

$\frac{q}{p}>0 \Rightarrow q>0$

From the graph [由圖中所見], slope of L_1 [L_1 的斜率] $>$ slope of L_2 [L_2 的斜率]

$-\frac{1}{m}>-\frac{1}{p} \Rightarrow m>p$

I is true. [I 為正確。]

x-intercept of L_1 [L_1 的 x 截距] $=n$ and [及] x-intercept of L_2 [L_2 的 x 截距] $=q$

From the graph, x-intercept of L_1 $<$ x-intercept of L_2

[由圖中所見，L_1 的 x 截距 $<$ L_2 的 x 截距]

$n<q$

II is true. [II 為正確。]

$n=-m \Rightarrow n+m=0$

$p>0$ and [及] $q>0 \Rightarrow p+q>0$

$n+m<p+q$

III is true. [III 為正確。]

10. By remainder theorem, the remainder of $(x+1)^{9394}+1$ divided by x equals to

$(0+1)^{9394}+1=1+1=2$

Put $x=24$ into $[(x+1)^{9394}+1]\div x$,

the remainder of $(25^{9394}+1)\div 24$ is 2

After $25^{9394}+1$ hours, the time will be 08:30.

$\therefore$ Dick Hui is awakening.

[根據餘式定理，$(x+1)^{9394}+1$ 除以 x 的餘數為

$(0+1)^{9394}+1=1+1=2$

代 $x=24$ 入 $[(x+1)^{9394}+1]\div x$ 中，

$(25^{9394}+1)\div 24$ 的餘數為 2

於 $25^{9394}+1$ 小時後，時間為 08:30

$\therefore$ Dick Hui 為清醒中。]

Suggested Solution [建議題解]

1. The answer is C

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$\angle MPN = \angle MQN = 80^\circ$

$\angle MBN = \dfrac{\angle MPN}{2} = 40^\circ$

$\angle MNC = \dfrac{180^\circ - \angle MCN}{2} = 70^\circ$

$\angle MAB = \angle MNC = 20^\circ + 50^\circ = 70^\circ$

$\angle MBA = \dfrac{180^\circ - \angle MAB}{2} = 55^\circ$

$\angle ABN = \angle MBA + \angle MBN = 40^\circ + 55^\circ = 95^\circ$

2. The answer is A

$\angle CAD = 180^\circ - 70^\circ - 60^\circ = 50^\circ$

$\angle OCD = 90^\circ$

$\angle OCA = 90^\circ - 70^\circ = 20^\circ$

$\angle OCA = \angle OAC = 20^\circ$

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$= 180^\circ - (2)(\angle OAC + \angle CAD)$

$= 180^\circ - (2)(20^\circ + 50^\circ)$

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From the graph [由圖中所見], slope of L_1 [L_1 的斜率] < 0

$-\frac{1}{m} < 0 \Rightarrow m > 0$

y-intercept of L_1 [L_1 的 y 截距] $= -1$

$\frac{n}{m} = -1 \Rightarrow n = -m$

$L_2 : x + py = q \Rightarrow y = -\frac{1}{p}x + \frac{q}{p}$

From the graph [由圖中所見], slope of L_2 [L_2 的斜率] < 0

$-\frac{1}{p} < 0 \Rightarrow p > 0$

y-intercept of L_2 [L_2 的 y 截距] > 0

$\frac{q}{p} > 0 \Rightarrow q > 0$

From the graph [由圖中所見], slope of L_1 [L_1 的斜率] $>$ slope of L_2 [L_2 的斜率]

$-\frac{1}{m} > -\frac{1}{p} \Rightarrow m > p$

I is true. [I 為正確。]

x-intercept of L_1 [L_1 的 x 截距] $= n$ and [及] x-intercept of L_2 [L_2 的 x 截距] $= q$

From the graph, x-intercept of L_1 $<$ x-intercept of L_2

[由圖中所見，L_1 的 x 截距 $<$ L_2 的 x 截距]

$n < q$

II is true. [II 為正確。]

$n = -m \Rightarrow n + m = 0$

$p > 0$ and [及] $q > 0 \Rightarrow p + q > 0$

$n + m < p + q$

III is true. [III 為正確。]

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[根據餘式定理，$(x+1)^{9394}+1$ 除以 x 的餘數為

$(0+1)^{9394}+1=1+1=2$

代 $x=24$ 入 $[(x+1)^{9394}+1]\div x$ 中，

$(25^{9394}+1)\div 24$ 的餘數為 2

於 $25^{9394}+1$ 小時後，時間為 08:30

$\therefore$ Dick Hui 為清醒中。]